HIGH-FREQUENCY APPROXIMATION FOR PERIODICALLY DRIVEN QUANTUM SYSTEMS IN THE VICINITY OF RESONANCES

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Intriguing dynamical quantum many-body effects such as prethermalisation, ergodicity breaking, and localisation observed in driven systems [1,2] are usually studied theoretically under the assumption that the external drive, $\hat{V}(t)$, is time-periodic. This allows one to employ the Floquet theory [3,4] and construct an effective time-independent Hamiltonian $\hat{H}_{\mathrm{eff}}$ that fully characterises dynamics of the system [5]. Although guaranteed to exist, effective Hamiltonians can be derived analytically in a few simple cases only. Otherwise, a high-frequency expansion proves useful, whereby $\hat{H}_{\mathrm{eff}}$ is constructed as an expansion in powers of $\hat{V}(t)/\omega$, where ω is the driving frequency [6]. This procedure becomes particularly transparent when formulated in the Floquet–Hilbert space, wherein the time-dependent Hamiltonian is mapped onto a time-independent one (called the Floquet Hamiltonian or the quasienergy operator), represented by an infinite matrix possessing block structure [3,4,7]. However, the high-frequency expansions currently described in the literature are inapplicable when the driving is resonant with transitions between states of the undriven system.

In this work, we derive a high-frequency expansion that treats the case of resonant driving. The derivation amounts to formulating a degenerate perturbation theory in the Floquet–Hilbert space and block-diagonalising the quasienergy operator to construct an approximation of $\hat{H}_{\mathrm{eff}}$. To demonstrate the validity of the derived expressions, we apply them to the driven Bose–Hubbard model [8] and calculate the quasienergy spectrum (eigenvalues of $\hat{H}_{\mathrm{eff}}$), which are physically significant. We perform calculations for different resonant conditions $nU = m\omega$ (U is the interaction strength of the Bose–Hubbard model; n and m are integers) and compare the results with numerically exact ones, which can only be obtained for small systems. The developed theory is seen to approximate the exact results in great detail.

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