

# APPLICATION OF MACHINE LEARNING TO NUCLEAR REACTIONS

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The Coulomb interaction between charged particles complicates the theoretical description of scattering even in a two body system because its long range asymptotic behavior does not satisfy the standard assumptions of scattering theory [1]. In practical calculations, the screened Coulomb interaction and renormalization method is often used [2,3], in which the unscreened potential

$$w(r) = \frac{\alpha Z_1 Z_2}{r} \quad (1)$$

is replaced by a short range form

$$w_R(r) = w(r) \exp[-(r/R)^n], \quad n \geq 1, \quad (2)$$

and the physical result is recovered by taking the limit  $R \rightarrow \infty$ .

Momentum space integral equation methods converge well at higher energies as the screening radius  $R$  is increased, allowing phase shifts to be computed with high accuracy. At lower energies, the influence of the Coulomb interaction becomes stronger and the numerical convergence with respect to  $R$  slows down, which reduces the accuracy of the calculations. This motivates the development of method that can reconstruct phase shift behavior at low energies using only a limited set of accurately computed values.

This work investigates the extrapolation of elastic two proton scattering phase shifts in the presence of the Coulomb interaction. The scattering problem is formulated within the Lippmann Schwinger integral equation and the  $t$  operator framework. The Coulomb interaction is included by using a screened Coulomb potential with screening radius  $R$  together with a renormalization procedure, and the extrapolation is performed with respect to  $1/R$  at fixed scattering energy  $E$ . The task is defined as reconstructing the phase shift curve toward the limit  $R \rightarrow \infty$  along decreasing  $1/R$  using only the first three points of the curve.

To solve this problem, a fully connected regression neural network is constructed. Instead of predicting the phase shift directly, the network predicts not the phase shift itself but the residual with respect to a simple linear baseline approximation, while additionally given local geometry information, namely the first and second derivatives. The model is trained on data generated within effective field theory and tested on an independent dataset obtained with the realistic CDBonn potential.

The results show that the mean relative error is largest at the lowest energies, reaching about 1–2%, and decreases with increasing energy, stabilizing around 0.2%. The obtained results show that the chosen neural network method is suitable for extrapolating phase shift curves beyond the range of known region, and it may be applied in the future to more complex systems, including three and four body nuclear reaction problems.

**Keywords:** Lippmann Schwinger equation, two proton scattering, screened Coulomb potential, phase shift, extrapolation, neural network, regression

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