DIFFUSION IN A QUADRATIC POTENTIAL WITH A CHANGING DIFFUSION COEFFICIENT

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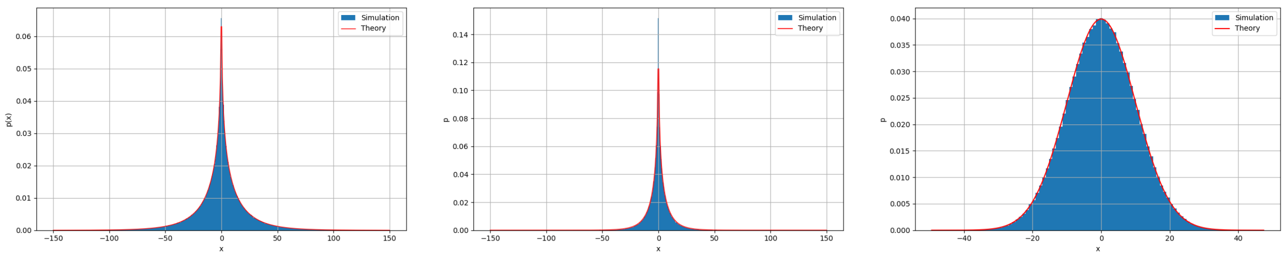
Diffusion with a changing diffusion coefficient can be used to model transport in heterogeneous media, such as living cells. In many systems, diffusing particles are confined to a limited region of space by an external potential. The simplest model for such confinement is a quadratic potential.[1]

In this work, we investigate two systems with a changing diffusion coefficient, described by systems of stochastic differential equations:

$$\begin{cases} dx_t = -\frac{1}{\tau_x}(x_t - \bar{x})\,dt + \sqrt{2D_t}\,dW_t^{(1)} \\ D_t = y_t^2 \\ dy_t = -\frac{1}{\tau_y} y_t\,dt + \sigma\,dW_t^{(2)} \end{cases} \quad \text{(1a)} \qquad \begin{cases} dx_t = -\frac{1}{\tau_x}\frac{D_t}{\bar{D}}(x_t - \bar{x})\,dt + \sqrt{2D_t}\,dW_t^{(1)} \\ dD_t = -\frac{1}{\tau_D}(D_t - \bar{D})\,dt + \sigma\sqrt{2D_t}\,dW_t^{(2)} \end{cases} \quad \text{(1b)}$$

Here, x_t represents the coordinate of a diffusing particle, and D_t represents the diffusion coefficient. Parameters τ_x, τ_y, τ_D, and σ determine the relative importance of the drift term and the noise term. In the system (1b), the drift term of the equation describing dx_t includes the diffusion coefficient. Similar systems were studied in [2].

For the first system we find the probability density functions for values of x_t in the long-time limit as well as the short-time limit. We compute time-dependent mean and variance of x_t. For the second system we find the PDF of x_t values in the long-time limit and approximate the mean and variance of x_t using multiple methods.



(a) (b) (c)

Fig. 1. (a) shows the long-time limit of the PDF of x values for the system (1a) with parameters $\tau_x = 1$, $\tau_y = 1000$, $\sigma = 1$, $\bar{x} = 0$, $T = 40$. T is the value of t when the simulation ended. (b) shows the short-time limit of the PDF of x values for the system (1a) with parameters $\tau_x = 1$, $\tau_y = 1000$, $\sigma = 1$, $\bar{x} = 0$, $T = 0.1$. (c) shows the long-time limit of the PDF of x values for the (1b) system with parameters $\tau_x = 1$, $\tau_D = 1000$, $\sigma = 1$, $\bar{x} = 0$, $\bar{D}$=100, $T = 30$.

These findings provide new insights into how varying diffusion affects confined systems and may inform future studies of complex stochastic dynamics

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[2] J. Ruseckas, R. Kazakevičius, and B. Kaulakys, “Coupled nonlinear stochastic differential equations generating arbitrary distributed observable with 1/f noise,” Journal of Statistical Mechanics Theory and Experiment, vol. 2016, no. 4, p. 043209, Apr. 2016, doi: 10.1088/1742-5468/2016/04/043209.